REVISTA INCLUSIONES

ESPACIO Y TIEMPO EN EL SIGLO XXI

Revista de Humanidades y Ciencias Sociales

Volumen 7 . Número Especial
Octubre / Diciembre
2020
ISSN 0719-4706

REVISTA INCLUSIONES M.R.
REVISTA DE HUMANIDADES Y CIENCIAS SOCIALES

CUADERNOS DE SOFÍA EDITORIAL

REVISTA INCLUSIONES M.R.
REVISTA DE HUMANIDADES Y CIENCIAS SOCIALES

CUADERNOS DE SOFÍA EDITORIAL

Editorial Cuadernos de Sofía
Santiago – Chile
OBU – C HILE

Indización, Repositorios y Bases de Datos Académicas

Revista Inclusiones, se encuentra indizada en:

CATÁLOGO

BIBLIOTECA UNIVERSIDAD DE CONCEPCIÓN

REVISTA INCLUSIONES M.R.
REVISTA DE HUMANIDADES Y CIENCIAS SOCIALES

CUADERNOS DE SOFÍA EDITORIAL

ISSN 0719-4706 - Volumen 7 / Número Especial / Octubre – Diciembre 2020 pp. 530-549

DIAGNOSTIC ASSESSMENT AND DEVELOPMENT OF LOGIC SKILLS IN FIRST-YEAR STUDENTS

Ph. D. Vladimir Yu. Tertychny-Dauri
Saint-Petersburg National Research University of Information Technologies, Mechanics and Optics (ITMO University), Russian Federation
ORCID: 0000-0003-4671-7659
tertychny-dauri@mail.ru
Ph. D. Ekaterina V. Milovanovich
Saint-Petersburg National Research University of Information Technologies, Mechanics and Optics (ITMO University), Russian Federation
ORCID: 0000-0002-9069-8574
milovanovich@mail.ru
Ph. D. Julia V. Tanchenko
Saint-Petersburg National Research University of Information Technologies, Mechanics and Optics (ITMO University), Russian Federation
ORCID: 0000-0003-2505-5158
jultan@mail.ru
Ph. D. Natalia V. Kondrashova
Saint-Petersburg National Research University of Information Technologies, Mechanics and Optics (ITMO University), Russian Federation
ORCID: 0000-0002-0322-8356
nvkondrashova@mail.ru
Ph. D. Yulia O. Valitova
Saint-Petersburg National Research University of Information Technologies, Mechanics and Optics (ITMO University), Russian Federation
ORCID: 0000-0001-5345-5461
julijawal@gmail.com
Ph. D. Anzhelika V. Morozova
Saint Petersburg Electrotechnical University "LETI", Russian Federation
ORCID: 0000-0003-1418-8000
morozova-spb@list.ru

Fecha de Recepción: 05 de junio de 2020 – **Fecha Revisión**: 20 de junio de 2020

Fecha de Aceptación: 27 de septiembre 2020 – **Fecha de Publicación**: 01 de octubre de 2020

Abstract

Today first-year university students have poor logic skills, which results in their equally poor performance in mathematics and low learning motivation. The aim of the present study was to carry out a diagnostic assessment among students to detect the level of their logic skills and to identify the conditions needed to develop them to the fullest extent possible. The respondents were 53 first-year university students who were proposed to take a test including five main types of mathematical problems. The research findings showed that the students were unable to argue and to prove due to the lack of oral skills. They had difficulty in providing the specification of a subject by factor, in identifying the similarities and differences of mathematical objects and subjects and many more. In view of the above, the authors' recommendations in respect of the intensification of oral activities in mathematics classes, summarized after the research study was over, will be in high demand not only among university, but also high school mathematics teachers because it is in high school that the learner's logic skills begin to develop.

Keywords

Logic skills – Diagnostics – Formation – First-years students – Logical thinking

Para Citar este Artículo:

Tertychny-Dauri, Vladimir Yu.; Milovanovich, Ekaterina V.; Tanchenko, Julia V.; Kondrashova, Natalia V.; Valitova, Yulia O. y Morozova, Anzhelika V. Diagnostic Assessment and Development of Logic Skills in First-Year Students. Revista Inclusiones Vol: 7 num Especial (2020): 530-549.

Introduction

Early 21st century marked a new stage in the development of society that highlighted the importance of values related to the unparalleled technological progress. New knowledge and technologies, on which the evolution of society in general largely depends, have started to come to the fore. Global changes in all spheres of life require the development of a new personality, i.e. a multi-faceted personality with a broad outlook capable of thinking actively and creatively, of quickly mastering the latest technologies and of acting independently while taking into consideration the constantly changing environment[1].

Unfortunately, present-day education does not meet the requirements laid down notably by society and production entities that need proactive and responsible employees who are ready to tackle a wide range of problems. The educational system needs to be revised and special emphasis should be put on the development of the learners' independence, their creative skills and their ability to implement the newly acquired knowledge in practice and in real life[2].

Today, Russian education sticks to competency-based learning aimed at teaching the learners knowledge and skills that may prove useful in their lives and further vocational education. This approach is practical and implies the development of key competencies in students.

Research into the numerous definitions of this notion provide an insight into what competency its: competency is the ability to make an active use of the acquired professional and personal knowledge during one's activity. A list of competencies can be found in the works of many Russian educationalists and psychologists, the most notable of which is the classification proposed by A. V. Khutorsky and I. A. Zimnyaya[3].

The former distinguishes between the cognitive, value-based and semantic, social, communicative, cultural, personal and information competences. As for Zimnyaya, she provides a broader view of key competences and divides them into three main groups that further include a specific set of skills:

1. Skills related to people as individuals and subjects of communication and activity.
2. Skills related to social interactions between people and their environment.
3. Skills directly related to human activities.

Thus, the viewpoints of Zimnyaya and Khutrosky about the classification of competencies imply that the learners' logical competency is also part of their key competencies.

Developing logical thinking is important not only to be a successful learner. Logic acts as a motivation icebreaker and fosters activity, values and many more. The current situation, however, is such that insufficient attention is given in high school to the learners'

---

[1] T. P. Varlamova, "Developing the Logical Competency in Grade 5 and 6 Students in Math Classes", Retrieved 28.08.2018 from: http://www.dissercat.com/content/formirovanie-logicheskoi-kompetentnosti-u-uchashchikhsya-5-6-klassov-v-protsesse-obucheniya-#ixzz5PLcs3v3p

[2] V. A. Krutetsky, The Fundamentals of Pedagogical Psychology (Moscow: Prosveshcheniye, 1996).

[3] Competency-Based Learning: Features, Issues and Interesting Facts. Retrieved 25.08.2018 from: http://fb.ru/article/43281/kompetentnostnyiy-podhod-v-obrazovanii

logical literacy and logical thinking despite their importance for the modern learner from the perspective of the competency approach. Later, this creates difficulties in their studying more advanced disciplines and, as a result, in the future employee's low professional competence.

Specialists in various fields have recently been found to be unable to perform their professional duties and to solve uncomplicated professional problems. Many believe that this has to do with high school education, more specifically, with high school and university students unwillingness to study mathematics.

In her research study on the psychological and pedagogical framework of high school teaching, psychologist L. F. Tikhomirova rightly notes that people do not think logically from birth. Instead, people acquire logical thinking skills throughout their lives and while in school[4]. N. F. Talyzina, N. A. Menchinskaya and N. B. Istomina consider that logical thinking is characterized by the ability to manipulate notions, judgments and speculations and its development is limited to the development of logical thinking techniques[5]. Despite different approaches to this issue, there is a consensus among researchers that the development of thinking operations – comparison, analysis and synthesis, generalization and classification – contributes to the general development of logical thinking.

Among researchers who studied the development of logical thinking by means of logic and mathematics were N. Y. Vilenkin, Yu. M. Kolyagin, N. A. Menchinskaya, T. N. Mirakova, I. L. Nikolskaya and many more. Researchers have repeatedly stressed the significance of studying logic for the development of logical thinking. A number of researchers worked on the learners' logical training as part of teaching mathematics, including V. M. Bradis, E. K. Voyshvillo, G. D. Gleyzer, G. V. Dorofeyev, A. N. Kolmogorov, N. I. Kondakov, P. S. Novikov, L. G. Peterson, N. F. Talyzina, A. I. Fetisov and B. V. Yakovlev.

As early as the 1990s, A. P. Kaloshina and G. I. Kharicheva said: "Difficulties that students have in studying mathematics are due to their insufficiently developed general logical thinking skills rather than to the complexity of the discipline itself"[6]. The faculty members of any higher education or vocational institution always find it difficult to teach various disciplines, including mathematics, because students have no logical thinking skills. First-year students are unable to define this or that notion and have problems with classifying entities by distinguishing characteristic and with proving their own or other's point of view. While learning terms and axioms by heart, they fail to grasp their meaning and to reflect on what they are talking about.

The spread of e-devices and their implementation into modern young people's lives aggravates the current situation even more. When solving a problem or doing an exercise in a math lecture or class, young men and girls follow the path of least resistance and call in the assistance of their electronic "friend", which decreases their learning motivation and disables their ability to think logically in order to obtain a result.

---

[4] L. F. Tikhomirova, Exercises for Every Day. Logic for Elementary Schoolchildren: A Popular Guidebook for Parents and Teachers (Yaroslavl: Akademiya razvitiya, Akademiya Co., 1998).
[5] V. I. Kurbatov, How to Develop Your Logical Thinking (Rostov-na-Donu: Feniks, 1997).
[6] A. P. Kaloshina, "Developing Logical Thinking Techniques", Sovetskaya pedagogika, num 4 (1975): 104-110.

In this context, the words of Russian scientist Mikhail Lomonosov about "the need to learn mathematics if for no other reason than to set the mind in order" are no longer relevant to modern students[7]. Mental activity is rapidly degenerating, resulting in unqualified staff being employed at the leading enterprises in the Russian economic, political and social sectors[8].

The above-mentioned issue calls for immediate intervention by researchers and practicing methodologists and research studies dedicated to the search for the best possible solution to this issue will be relevant for a long time to come.

In this regard, the main objective of this study is not only to analyze the level of first-year students' logical competency, but also to propose how to improve the very process of acquiring logical thinking skills.

The subject of inquiry, in this case, are first-year students

The subject of research is the level of logical literacy or logical thinking in first-year students.

The research hypothesis is the implementation of oral assignments and exercises in teaching university mathematics courses by bringing back oral examinations and colloquia on major topics, saying out loud the entire problem-solving process, focusing on the students' ability to compare different mathematical notions, singling out their characteristic features, etc.). The above measures will improve the development of first-year students' logical thinking skills.

To achieve the goal, the following steps are to be taken:

- To test students to determine the level of their logical competency;
- To study and summarize the test results;
- To suggest measures aimed at improving the students' logical thinking skills.

Methods

The ITMO University carried out a research study on the logical skills and high school mathematical competencies as part of the national program No. 3407 entitled *Methodological Guidance on the Implementation of a Mathematical Education Concept in the Russian Federation in 2015*[9]. As part of this study, students were given a short questionnaire containing a number of logical and mathematical problems with a view to detect the level of the learners' verbal and logical thinking skills, perceptions and memory.

The test aimed to determine the prospective students' ability to think logically, to find and establish causal relationships between entities, to identify the essentials and to classify

---

[7] E. V. Yakovleva, "Significance of Students' Psychological Mental Specificities in the Development of Their Logical Culture", Obshchestvennye nauki. Pedagogika, num 1 (2007): 63-71.

[8] N. Bulgakov, "The Present-Day Degeneration of the Brain". Retrieved 28.08.2018 from: https://www.ranibu.ru/sovremennaya-degradatsiya-mozga.html

[9] E. V. Milovanovich; I. V. Seyfer & Yu. V. Tanchenko, "Logical Skills and School Math Competencies". Pedagogika i psikhologiya: tendentsii i perspektivy razvitiya. Sbornik nauchnykh trudov po itogam mezhdunarodnoy nauchno-prakticheskoy konferentsii, Volgograd, 10 October, 2015. Vol: II (2015): 170-172.

objects by distinguishing characteristic. Practice shows that the absence of these qualities in students considerably complicates the process of teaching advanced mathematics and/or calculus to them. Furthermore, upon completion of the present study, the authors would like to suggest a list of recommendations directed at eliminating difficulties and obstacles in the development of logical competency among first-year students.

The study adopted the following research methods:

1) A theoretical analysis of relevant teaching and training materials with a view to study the current state of the topic under investigation and its methodological framework.
2) An analysis of research works in order to elucidate the scope of issues related to the research topic.
3) Organization and conduct of the ascertaining, exploratory and forming experimental investigations.
4) Statistical processing of the data obtained during the experimental investigation.

The research study was divided into three stages. First, relevant research literature was studied and research objectives were formulated. Then, problems were selected for the forthcoming testing, the experiment itself was carried out and, finally, the thoroughly processed and interpreted results were presented.

The reliability of research outcomes is due to the in-depth study of academic works related to the research topic, to the research methods corresponding to the research objectives, to the correct conduct of the experiment itself and of data processing and, finally, to the confirmation of the suggested hypothesis using the test results.

The testing involved three groups of students. The first, strong, group (advanced undergraduate students enrolled in the Advanced Mathematics and Computer Science program) consisted of government-subsidized students whose GPA was 81 (Unified State Examination). The second, average, group consisted of mostly government-subsidized Computer Technology and Management students whose GPA was 67. The third, weak, group consisted of contracted Business and Computer Science students whose GPA was 52.9. The testing time was 40 minutes.

The students were proposed a number of mathematical problems aimed at activating their mental ability. The problems were divided in the following groups:

a) Problems designed to replication (their solution involves mathematical memory and attention);
b) Problems whose solution led to a new idea;
c) Creative problems.

Thus, the testing comprised word problems; problems that involved logical reasoning, associative thinking, proof development, generalization and detection of attributes, finding errors in a math proof and problems for deeper understanding of mathematical concepts.

Researchers propose different definitions of the notion of ‘problem’. Gurova focuses on the object of the mental efforts of a person solving a problem: “A problem is the object of mental activity that requires a practical transformation or an answer to a theoretical question by searching for conditions that reveal relations between its known and unknown elements”[10]. Ball suggests the following definition: “In its most basic form, a problem is a system with the following mandatory components: a) the subject matter of the problem in its initial state; b) the model of the required state of the subject matter of the problem (this model is identified with the requirements of the problem”[11]. A standard structure, however, unites all types of problems, that is, each of them has a statement, a rationale, a solution and a conclusion.

Thus, the reason why word problems were used in the test was to verify how well the learners can identify the gist of the problem, make a plan to solve it and to translate the problem’s wording into the language of mathematics.

A *word problem* is the description of a problem or a problematic situation in the conventional language, requiring to provide a quantitative characteristic of this or that situational component[12]. This is the most widespread type of mathematical problems, also known as arithmetic or real-world problems. Given that these problems demand the learner to find the unknown and are limited to compute the unknown value, they are also called computational problems.

Much attention is focused on word problems because they are of great importance for brain building.

Logic problems. Logical reasoning stimulates the activity of all components of knowledge assimilation, i.e. attention, memory and imagination, which is the basis of the learners’ intelligence. Correspondingly, it develops the learners’ logic skills[13]. Logic problems are assessed as having high potential. They develop the learners’ critical assessment skills, help them analyze the learning material and increase their general motivation to study mathematics. The didactic value of such problems is indisputable.

In solving logic problems, people learn to reason. The absence of specific knowledge that could lead to the desired result makes the learner to draw on his own arguments. In the end, by solving such problems, the student learns to express his idea and prove it in a well-argued manner and uses logical reasoning to solve the problem.

In these problems, reasoning appears as abstract notions and arguments that reflect the essential aspects of the surrounding reality and the regular links between them. These problems develop abstract and theoretical thinking and set no limit on expanding mathematical knowledge.

Associative problems. The associative aspect of problems has to do with the ability to see various forms in their spatial and flat representations, to identify configurations, to imagine what the graph of a function looks like, to know the properties of a function and to relate figurative ideas with identity transforms. The associative aspect develops associative

---

[10] N. A. Khitrina, “On Using Counter-Examples”, Matematika v shkole, num 6 (1974): 8-14.

[11] N. L. Stefanova, Methods of Teaching Mathematics: A Series of Lectures. Stefanova, N. S. Podkhodova et al. (Moscow: Drofa, 2007).

[12] Yu. M. Kolyagin, Problems in Teaching Mathematics. Chapter 1: Mathematical Problems as a Means for Teaching and Developing Learners (Moscow: Prosveshcheniye, 1977).

[13] E. M. Vechtomov & Ya. Petukhova, “Solving Logic Problems as a Basis for Developing Thinking”, Kontsept, num 8 (August) (2012). Retrieved 28.08.2018 from: http://www.covenok.ru/koncept/2012/12109.htm

thinking and help feel the wholeness of entities under investigation. Associative thinking helps accumulate new knowledge in the peripheral areas of science and to identify problems that do not belong explicitly to this or that discipline. In solving problems, associative thinking contributes to the development of memory and attention.

Proof construction math problems. Proof construction problems exert a significant influence on the development of the learners' thinking skills. When solving construction problems, the learners improve their logic thinking skills, find logical ways to solve problems and feel the need to substantiate mathematical facts[14].

Geometric problems develop the learners' algorithmic thinking skills, analytical and synthetic activity as well as flexible and constructive thinking. Proof construction problems improve the learners' analytical thinking, skills in searching for ways to solve practical problems and introduce them to independent research activity commensurate with their abilities. Proof construction problems let the learners gain a better understanding of the theoretical knowledge about basic geometric figures.

Finding errors in a math proof. Researchers Zaykina, Grigoryeva and Subbotina suggested that thought-provoking problems may be used to avoid errors in learning mathematics[15]. In other terms, such problems contain an error that can be found only in the problem-solving process. Math problems containing an error in their proof are one of the varieties of thought-provoking problems aimed at developing oral skills.

Students should be reminded that bad oral skills often pose a serious obstacle to the assimilation of mathematical knowledge and result in incorrect statements, hence the need to include this type of problems in the mathematics curriculum since they develop the learners' critical thinking skills and help them distinguish between closely related notions. The material was designed with special consideration for the objective of the testing as well as the age-related and psychological characteristics of the respondents.

Thinking specification problems. It is important for first-year students not to lose another component of logical reasoning, namely, its specification and detailing, in the process of learning and formation of abstract, associative and generic critical thinking skills. The point at issue is the formation of analytical skills concerning the splitting of mathematical notions into specific parts and components interconnected by causal relationships. As an example, the notions/definitions of limit and Cauchy-continuous function are to be specified with illustrations showing the relationship expressed in the epsilon-delta language between arguments and functions. Similar considerations relating to the visual/illustrative specification of component parts for various mathematical notions can be given to define the notions of integral, stability and integral curves, among others.

Results

The students were given seven types of problems in two versions. The versions of the tests can be found in Annex 1. The research results for all groups have been presented as a table and a diagram.

---

[14] Yu. A. Sherashova, "Solving Proof Problems as a Step towards Development of Logical Thinking", Retrieved 28.08.2018 from: http://открытыйурок.рф/статьи/213176/

[15] M. I. Zaykin & V. A. Kolosova, "Thought-Provoking Problems as a Means for Developing Critical Thinking in Schoolchildren", Nachalnaya shkola, num 9 (2002): 73 – 77.

Problems	**Version 1 (4 students) %**	**Version 2 (4 students) %**	**Overall %**
1	25	100	62.5
2	100	50	75
3	50	100	75
4	75	25	50
5	75	50	87.5
6	0	0	0
7	25	50	37.5

Table 1
Results of the logic competency test taken by first-year students from Group 1 (the strong group)

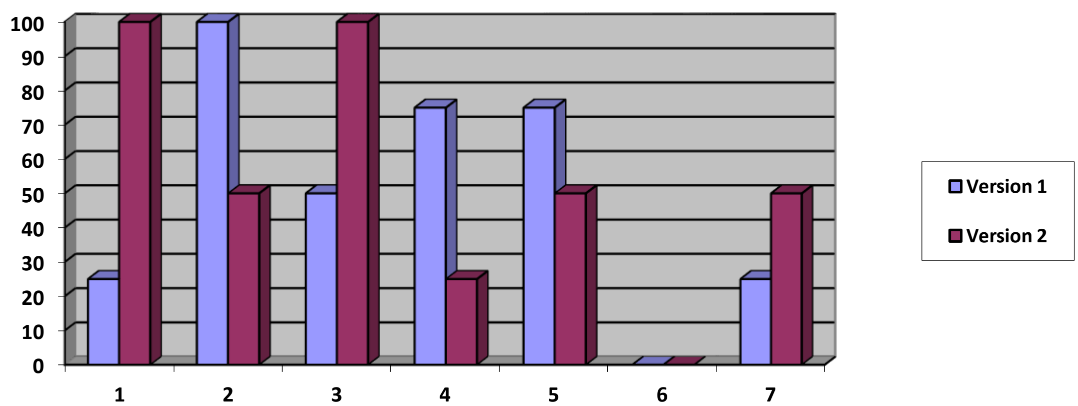


Fig 1
Based on Table 1 for both test versions (Group 1)

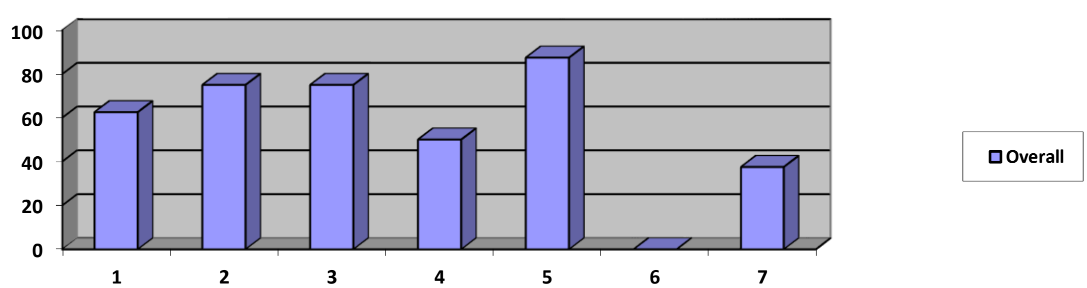


Fig. 2
Shows the overall results for both test versions (Group 1)

Problems	Version 1 (12 students) %	Version 2 (7 students) %	Overall %
1	33	85	53
2	83	71	79
3	66	86	73
4	25	14	21
5	75	14	53
6	17	14	16
7	25	0	16

Table 2
Results of the logic competency test taken by first-year students from Group 2 (the average group)

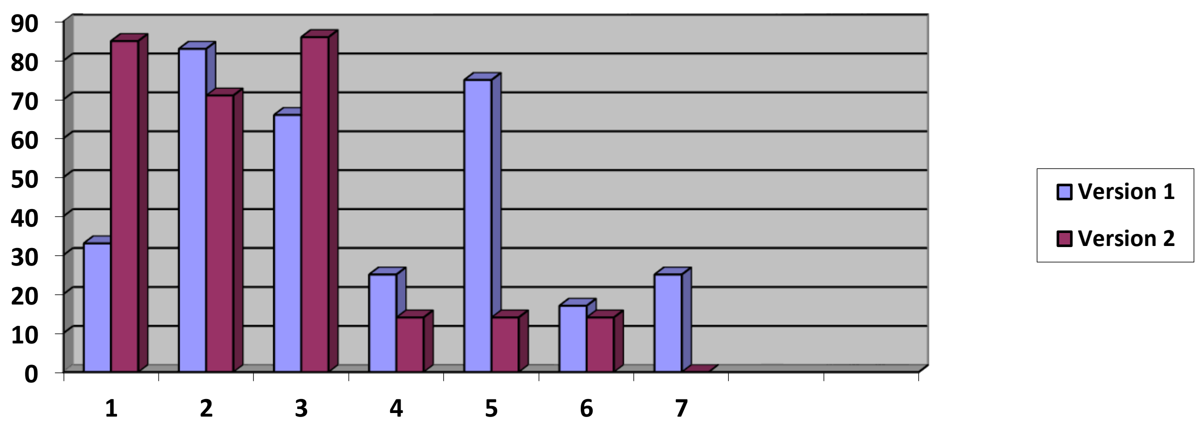


Fig. 3
Based on Table 2 for both test versions (Group 2)

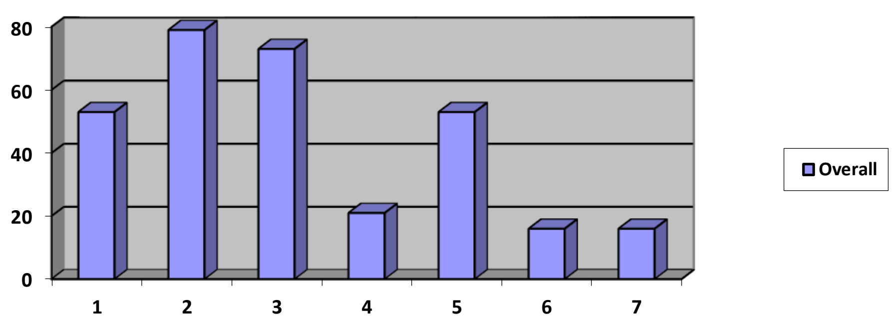


Fig. 3
Shows the overall results for both test versions (Group 2)

Problems	**Version 1 (13 students) %**	**Version 2 (13 students) %**	**Overall %**
1	0	77	38
2	69	61	65
3	7.7	100	53.8
4	23	15.5	19.2
5	69	0	34.5
6	7.7	0	3.85
7	0	0	0

Table 3
Results of the logic competency test taken by first-year students from Group 3 (the weak group)

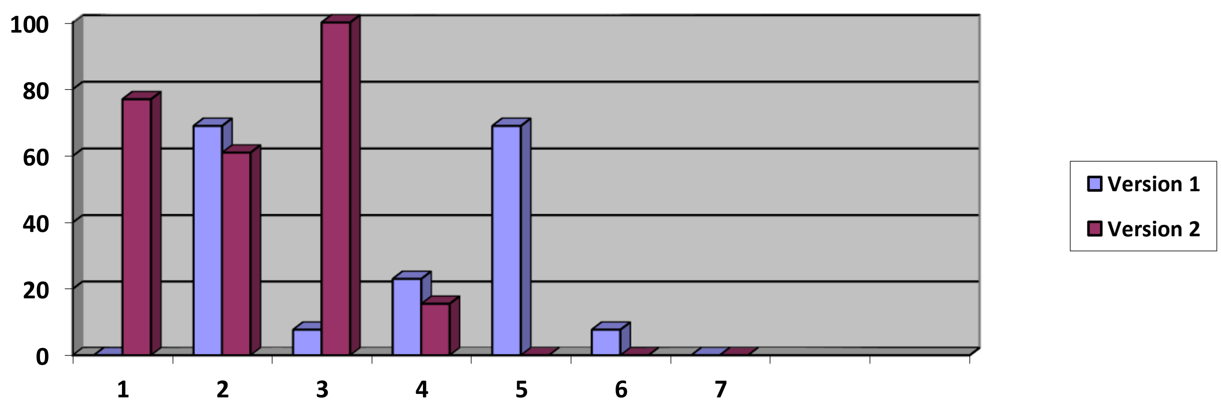


Fig. 5
Based on Table 3 for both test versions (Group 3)

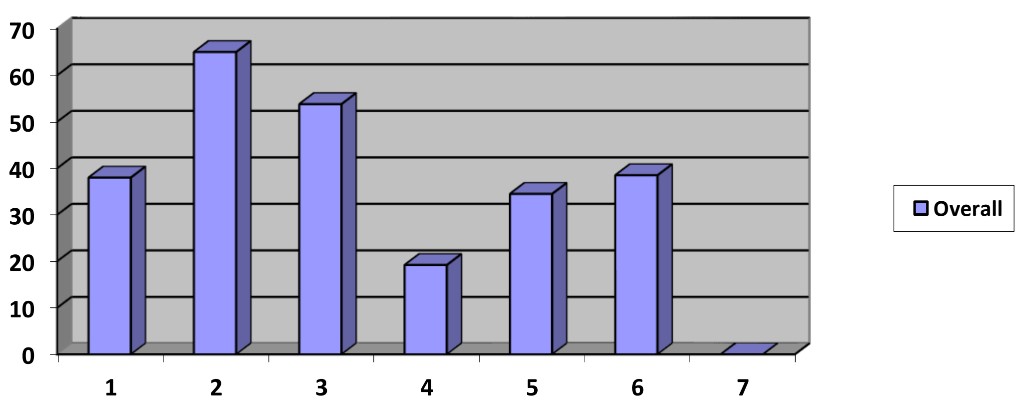


Fig. 6
Showing the overall results for both test versions (Group 3)

Discussion

Our analysis of the results led to the conclusion that most first-year students had very poor logical thinking. This was mostly due to the Unified State Exam (USE) in Mathematics. Multiple-choice questions allow students to tick the box randomly without making efforts to solve problems and equations.

It is also worth mentioning that high school students do not usually learn poems by heart (therefore, they have problems with memory). In algebra classes, they solve only standard problems, for which they have to find an appropriate formula and put the necessary data into it (consequently, they have problems with logic and intuition). Finally, they do not write geometry proofs in geometry classes, hence problems with theoretical thinking, analytical skills, synthesis and verbal communication. The decreasing class time is mostly given to the primitive training of high school students aimed at preventing them from failing the USE and, thus, disgracing their school.

It is not uncommon for high school graduates who passed the USE in Russian language and Mathematics to be unable to write correctly and to solve a simple, but unconventional problem, respectively. Even students with high – and sometimes very high – USE score in Mathematics cannot clearly express the definitions of major mathematical notions and have difficulty in explaining steps in solving a correctly defined problem. Their attempts at substantiating their actions are often reduced to flailing hand gestures and interjections. This is why the aim of the given testing was not so much to confirm once again that the absolute majority of first-year students lacked the ability reflect and argue as to find ways to deal with this issue and to take timely measures to create an effective learning environment.

Our analysis of the results detected several widespread mistakes made by almost all participants in the experiment:

1. Students regard as axioms Pythagoras' theorem and that of the sum of the angles of a triangle.
2. Students regards as theorems the axioms of set theory and the parallel axiom.
3. Many students do not see the difference between axioms and theorems.
4. The third group of students did not even get to the last problem.

The results for the Version 2 of the third assignment stand out from other results. Only one person did not manage to give the correct answer, whereas many students failed to give the correct answer to a similar assignment from Version 1.

In the above-mentioned assignment, students were given five words and had to find the redundant one. In Version 1, the words proposed were *force, weight, tension, mass* and *velocity*. The redundant word is *mass* because it is a scalar quantity, whereas the remaining ones are vector quantities. In Version 2, the words proposed were *second, hour, year, morning* and *month*. The redundant word is *morning*, since it is not a unit of time, unlike others. Interestingly, the student who, according to the examiner, failed to give the correct answer had been right, in a sense. In his opinion, the redundant word was *month* because the number of components (days) changes in months, whereas the number of components is invariable in other notions.

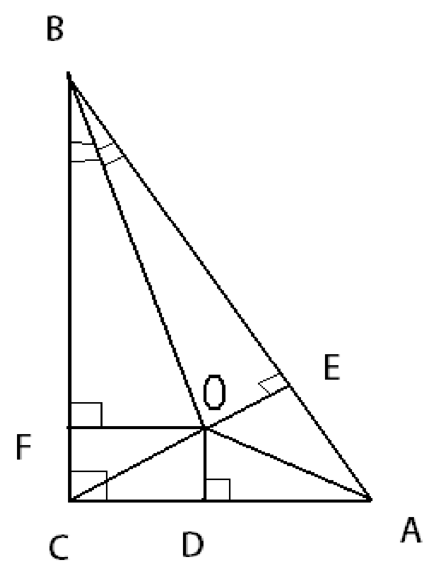
In any case, more students answered correctly the question from Version 2 than from Version 1. It may be due to the fact that the notions from Version 2 are taken from everyday life and those from Version 2 are taken from physics and require deeper knowledge of this discipline.

Examiners were unpleasantly surprised that some students with a high USE score who graduated from high-ranking high schools and who were planning to continue studying mathematics did not know the basic geometry axioms and were unable to write simple geometry proofs.

Students were exceptionally bad at trying to solve the last problem on finding errors in a math proof. Only 11% of the total number of the respondents solved it correctly, that is, four students for Version 1 and two students from all groups for Version 2.

Of special interest is the discussion of a problem from *Errors in Mathematical Reasoning*, a book by M. V. Bradis[16]. The objective was to refute the proof of the fact that the leg is equal to the hypotenuse in a right triangle. Only two students from the average group (in terms of USE scores) solved the problem correctly. Below is the statement of the problem.

Given: ABC is a right triangle, AB is a hypotenuse, BO is an angle bisector of B, OD is a perpendicular bisector of AC. Find AB=BC.



Proof:

1. OF= OE because the points lying on the bisector of the angle are equidistant from its sides. VO is the common side of triangles BFO and BEO. Therefore, triangle BFO is equal to triangle BEO along the leg and the hypotenuse. Therefore BF=BE.

[16] V. M. Bradis; V. L. Minkovsky & A. K. Kharcheva, Errors in Mathematical Reasoning (Moscow: Gosudarstvennoye uchebno-pedagogicheskoye izdatelstvo ministerstva prosveshcheniya RSFSR, 1959)

2. OC=OA because all points of the perpendicular bisector are equidistant from its ends. Therefore, triangle CFO is equal to triangle AEO along the leg and the hypotenuse. Therefore CF=AE.
3. BF=BE, CF=AE. Therefore, BC=AB. Proved.

The testing outcomes point to the need to revise the methods of teaching mathematics in high school because it lays the groundwork for basic logical thinking. According to Vygotsky and Piazhe, the most propitious time to start developing children's logical/theoretical thinking skills is at age 11 to 12. Davydov, Peterson and Elkonin, among others, are confident that the potential mental capacities of schoolchildren from this age group are wider and richer than those that are traditionally developed[17].

Traditional methods are no longer relevant today. Their focus on the reproduction and repetition of information makes them incapable of uncovering the potential of students. In this context, developing the students' logical thinking and, generally speaking, mental activity is out of question. Prospective students entering the higher education system, especially technology students, face a number of challenges that badly affect their academic performance.

The first-year students' poor performance is evident in the results of the testing aimed at revealing the level of their logical literacy. The average and low average score point to the need to spend more time on developing children's independence, critical thinking, argument and reasoning skills while they are still in high school. In short, they should talk more in their Math classes.

Conclusion

The aim of higher education is to train qualified specialists, hence a list of requirements for undergraduate students. The first year of college is the most difficult and demanding one because the learner moves to a higher stage of education. An enhanced flow of information, insufficient time allotted to close examination of this or that topic, the complexity of the study material create certain difficulties for Math students.

Besides, every discipline sets its own requirements. As an example, the study of mathematics is aimed at providing students with mathematical methods of reasoning and at broadening their knowledge in mathematics to meet the needs of society[18]. However, the lack or underdevelopment of basic logical thinking skills in students makes their first year of college difficult.

The results of the testing lead to the conclusion that students enrolled in a two-year advanced mathematics program showed poor performance in mathematics while at high school and, moreover, thy have almost no skills in proving, summarizing, systematizing mathematical contents or in building a line of reasoning.

[17] R. A. Atakhanov "A Diagnostic Assessment of the Development of Mathematical Thinking". Voprosy psikhologii, num 1-2 (1992): 60-67.

[18] T. P. Kuryachenko "Difficulties Faced by First-Year Students in Studying the Limit at a Point in Calculus Courses". Matematika i informatika: nauka i obrazovaniye: Mezhvuzovsky sb. nauch. trudov: Yezhegodnik, issue 5 (2006) 154-157.

Importantly, simple logical problems are within the capacity of even weak students (thus, even those students whose GPA was 33 in the USE in Mathematics were able to solve the problem No. 2).

In this context, the educational process should be organized in a way that activates students' mental activity as much as possible. Unfortunately, there is no uniform view on this point. Some educators believe that logical thinking develops automatically during the study of all disciplines included in the general education curriculum (Beylinson, Pospelov, Skatkin)[19].

Other researchers insist on the impossibility to grasp the basic laws of logic using only study disciplines, hence the urgent need to include logic courses in the high school curriculum (Vering, Lifintseva, Nurgaliyev, Palamarchyuk)[20].

In any event, learners should be introduced to logic during their high school years, and the following measures might prove effective for development logical skills in first-year students:

- Bring back oral examinations and colloquiums on major topics to the advanced mathematics program and add hours to the curriculum for this purpose;
- Set aside more seminar time for short and simple theorems that are part of the linear algebra program (for instance, properties of linearly dependent vectors) and of calculus (necessary and sufficient criterion for extreme points or Lagrange theorem);
- Make an extensive use of the inverse problem method (closely examine the proof of a theorem and then suggest that students formulate it on their own);
- Use the most straightforward and explicit problems when starting a topic and make students pronounce orally at every solution step. Visually illustrating the problem solving process incites students to reason, i.e. to adopt logical research means contributing to the development of mental operations.
- Suggest students to determine the similarities and differences between these or those mathematical objects by identifying their distinctive characteristics (matrix and vector, vector and segment, definite and indefinite integral);
- Provide students an oral description of a mathematical object and then suggest they find a definition for this object;
- Use different types of problems in class and allow first-year students to solve the independently, thus fostering their creativity and logical skills.

In the authors' view, these measures will improve the current situation, will facilitate university students' understanding and assimilation of mathematical knowledge and will contribute to further development of logical competency among first-year students. In his *Prelude to Mathematics*, Sawyer observes that "students can be taught many different types of problems, but the real satisfaction comes only when we can transmit to them not only knowledge but also the flexible mind", which would allow them, in the future, to solve problems on their own and set new problems to themselves[21].

---

[19] N. B. Istomina, Methods of Teaching Mathematics in Elementary School: A Textbook (Smolensk: Izd-vo Assotsiatsiya XXI vek, 2005)

[20] O. S. Goncharova "Developing Logical Thinking in Math Classes in Elementary School". Molodoy uchyony, num 10 (2012): 329-331.

[21] W. W. Saywer, Prelude to Mathematics (Moscow: Prosveshcheniye, 1965).

To conclude, the authors would like to highlight that the objectives set at the beginning of the study have been accomplished successfully:

1. An analysis of available research works revealed different points of view on the development of logical thinking. Most researchers, however, agree that high school years are the most propitious time to start developing logical skills. During this period, the learner's mental processes should be activated as much as possible in math classes with a view to provide a solid foundation for further study of this discipline.
2. The choice of the testing content (problems) was made with due regard to the main goal of the inquiry, i.e. to determine the development of logical skills among first-year students.
3. The low results of the testing conducted among first-year students with a view to determine their logical competence proved the inadequacy of current methods of teaching mathematics and gave the authors an idea of the need to seek better ways to deal with this issue.
4. The data obtained confirmed the working hypothesis put forward at the beginning of the study, i.e. the lack of "talking" in math classes results in the impoverishment of the learners' mathematical vocabulary, in poor articulation of their thoughts and in underdeveloped skills in argumentative and logical discourse. This is why an active use of verbal methods in first-year mathematics courses will broaden the students' opportunities in logical thinking, thus helping them gain deeper mathematical knowledge.

The finding of the study can be used as a theoretical framework for further research on the development of logical skills in students taking advanced mathematics courses. Besides, math teachers in high school may use this information as a helpful guideline when working on their curricula.

References

Atakhanov, R. A. "A Diagnostic Assessment of the Development of Mathematical Thinking". Voprosy psikhologii, num 1-2 (1992): 60-67.

Bradis, V. M.; Minkovsky, V. L. & Kharcheva, A. K., Errors in Mathematical Reasoning. Moskva: Gosudarstvennoye uchebno-pedagogicheskoye izdatelstvo ministerstva prosveshcheniya RSFSR. 1959.

Bulgakov, N. "The Present-Day Degeneration of the Brain". Retrieved 28.08.2018 from: https://www.ranibu.ru/sovremennaya-degradatsiya-mozga.html

Competency-Based Learning: Features, Issues and Interesting Facts. Retrieved 25.08.2018 from: http://fb.ru/article/43281/kompetentnostnyiy-podhod-v-obrazovanii

Goncharova, O. S. "Developing Logical Thinking in Math Classes in Elementary School". Molodoy uchyony, num 10 (2012): 329-331.

Istomina, N. B. Methods of Teaching Mathematics in Elementary School: A Textbook. Smolensk: Izd-vo Assotsiatsiya XXI vek. 2005.

Kaloshina, A. P. "Developing Logical Thinking Techniques". Sovetskaya pedagogika, num 4 (1975): 104-110.

Khitrina, N. A. “On Using Counter-Examples”. Matematika v shkole, num 6 (1974): 8-14.

Kolyagin, Yu. M. Problems in Teaching Mathematics. Chapter 1: Mathematical Problems as a Means for Teaching and Developing Learners. Moscow: Prosveshcheniye. 1977.

Krutetsky, V. A., The Fundamentals of Pedagogical Psychology. Moscow: Prosveshcheniye. 1996.

Kurbatov, V. I. How to Develop Your Logical Thinking. Rostov-na-Donu: Feniks. 1997.

Kuryachenko, T. P. “Difficulties Faced by First-Year Students in Studying the Limit at a Point in Calculus Courses”. Matematika i informatika: nauka i obrazovaniye: Mezhvuzovsky sb. nauch. trudov: Yezhegodnik, issue 5 (2006) 154-157.

Milovanovich, E. V.; Seyfert, I. V. & Tanchenko, Yu. V. “Logical Skills and School Math Competencies”. Pedagogika i psikhologiya: tendentsii i perspektivy razvitiya. Sbornik nauchnykh trudov po itogam mezhdunarodnoy nauchno-prakticheskoy konferentsii, Volgograd, 10 October, 2015. Vol: II (2015): 170-172.

Saywer, W. W., Prelude to Mathematics. Moscow: Prosveshcheniye. 1965.

Sherashova, Yu. A. “Solving Proof Problems as a Step towards Development of Logical Thinking”. Retrieved 28.08.2018 from: http://открытыйурок.рф/статьи/213176/

Stefanova, N. L. Methods of Teaching Mathematics: A Series of Lectures. Stefanova, N. S. Podkhodova et al. Moscow: Drofa. 2007.

Tikhomirova, L. F. Exercises for Every Day. Logic for Elementary Schoolchildren: A Popular Guidebook for Parents and Teachers. Yaroslavl: Akademiya razvitiya, Akademiya Co. 1998.

Varlamova, T. P. “Developing the Logical Competency in Grade 5 and 6 Students in Math Classes”. Retrieved 28.08.2018 from: http://www.dissercat.com/content/formirovanie-logicheskoi-kompetentnosti-u-uchashchikhsya-5-6-klassov-v-protsesse-obucheniya-#ixzz5PLcs3v3p

Vechtomov, E. M. & Petukhova, Ya. “Solving Logic Problems as a Basis for Developing Thinking”. Kontsept, num 8 (August) (2012). Retrieved 28.08.2018 from: http://www.covenok.ru/koncept/2012/12109.htm

Yakovleva, E. V. “Significance of Students’ Psychological Mental Specificities in the Development of Their Logical Culture”. Obshchestvennye nauki. Pedagogika, num 1 (2007): 63-71.

Zaykin, M. I. & Kolosova, V. A. “Thought-Provoking Problems as a Means for Developing Critical Thinking in Schoolchildren”. Nachalnaya shkola, num 9 (2002): 73 – 77.

Annex 1
Test, Version 1

Content of the Problem	Solution of the Problem	Rationale
1. Kolya's stamp collection consists of three albums. Two tenths of his stamps are in the first album, several sevenths in the second album and 303 in the third one. How many stamps does Kolya have? (Is the statement sufficient to determine the variable?)	An equation is written, where x are stamps, y are several sevenths $\frac{2x}{10}+\frac{yx}{7}+303=x$, the denominator would be positive and uneven because it must divide the numerator which is uneven. This gives three possibilities: y = 1, 3 and 5, only y = 5 gives the divider for the numerator. Therefore, y = 5, x = 3535. The answer is 3535 stamps. The statement is sufficient.	This problem develops abstract and logical thinking, helps learners to identify the essential properties of the object and prepares them to solve equations with multiple variables.
2. There are two similar boxes on the table. In each of them is either a white ball or a black one. One of the boxes has the inscription: "In this box there is a white ball and in the other box there is a black ball". The other box has this inscription: "In one of these boxes there is a white ball; besides, a black ball is in one of these boxes". One of the inscriptions is known to be true and the other one false. Is there a white ball in one of the boxes and, if yes, in which one?	The first inscription means that the white ball is in the first box and the black one in the second box. The second inscription means that the white ball is in the 1st box, the black one is in the 2nd box or the black one in the 1st box and the white one in the 2nd box. Let us examine two possible options. A) The first inscription is true and the second one false. B) The first inscription is false and the second one is true. Option A is contradictory, so Option B is the right one. Answer: the white ball is in the second box.	Develops logical thinking and checks learners' knowledge of basic laws of logic, logical reasoning and conclusions.
3. Which notion do you think is redundant? Why? Justify your answer: **Force, velocity, weight, mass, tension.** 4. Prove that diagonal lines, the end ones being equal, can divide the median of a trapezoid in three segments.	The redundant notion is mass because it is a scalar quantity and the remaining ones are vector quantities. The extreme segments of the mean line of the trapeze are the mean lines of two triangles with a common third side. Therefore, they are equal and equal to its half.	This problem checks learners' skills in summarizing and classifying objects, in identifying a common feature (the foundation of classification) and in discarding the superfluous. This theoretical thinking problem develops learners' logic skills and ability to reason, to analyze and to summarize. Learners'

		proficiency in providing proofs is of great importance in the study of advanced mathematics and, especially, of calculus.
5. In which case is the quadrilateral ABCD not a parallelogram? a) AB is not parallel to CD or BC is not parallel to AD b) AB is not parallel and equal to CD c) AB is not parallel to CD or is not equal to CD d) AB is not parallel to CD and BC is not parallel to AD e) AC is not equal to BD or AC is not perpendicular to BD	The given quadrilateral is a parallelogram only in the last case. In the last case, the quadrilateral is neither a rectangular nor a rhombus, i.e. it is not a particular parallelogram case, but it may be an arbitrary parallelogram.	This problem tests learners' knowledge of the foundations of plane geometry and of the properties of parallelograms. Since parallelograms are most frequent in vector algebra and analytic geometry courses, learners have to know the properties of parallelograms and to be able to find its parameters in order to pass these courses. Furthermore, this problem prepares learners for the study of the basic logical operations of conjunction, disjunction and negation and teaches them to express the negation of a statement.
6. Which ones of the statements below do you think are axioms and which ones are theorems? a) The sum of the angles of a triangle is equal to 180^0. b) Each segment has definite length. c) Only one plane can be drawn through three points lying on a straight line. d) A line is perpendicular to a plane if it is perpendicular to every line, contained in the plane, passing through the point of intersection. e) When a transversal line intersects two parallel lines, the sum of the two one-sided angles is 180^0. f) A straight angle is equal to 180^0. j) The square of the hypotenuse is equal to the sum of the squares of the other two sides. 7. A student provided the following proof of the equation below: $\sqrt[3]{5\sqrt{2}+7}-\sqrt[3]{5\sqrt{2}-7}=2$. « If both sides of the equation are raised the third power, we get	b, c and f are axioms. Other statements are theorems. The proof is wrong. The error comes after the words "However, given that" $\sqrt[3]{5\sqrt{2}+7}-\sqrt[3]{5\sqrt{2}-7}=2$, $\sqrt[3]{(5\sqrt{2}+7)}\cdot\sqrt[3]{5\sqrt{2}-7}=1$ Here, the learner uses a fact that is yet to be proved. Consequently, his further reasoning is erroneous.	This problem tests learners' knowledge of basic geometry laws. Learners should understand that it is impossible to prove everything and that some statements are taken as true in order to base further reasoning on them. This problem prepares learners to understand the axiomatic structure of a number of mathematical disciplines and to distinguish between axioms and theorems. This problem develops learners' critical and analytical thinking and curiosity. It also teaches learners to analyze, to compare and to argue, while developing their skills in setting and solving problems correctly and in

$14-3(\sqrt[3]{(5\sqrt{2}+7)^2}\cdot\sqrt[3]{5\sqrt{2}-7}$ - $\sqrt[3]{5\sqrt{2}+7}\ \sqrt[3]{(5\sqrt{2}-7)^2}$)=8 Therefore, 3 $\sqrt[3]{(5\sqrt{2}+7)}\cdot\sqrt[3]{5\sqrt{2}-7}$($\sqrt[3]{5\sqrt{2}+7}$ - $\sqrt[3]{5\sqrt{2}-7}$)=6 However, given that $\sqrt[3]{5\sqrt{2}+7}-\sqrt[3]{5\sqrt{2}-7}=2$, $\sqrt[3]{(5\sqrt{2}+7)}\cdot\sqrt[3]{5\sqrt{2}-7}=1$ We take the root of the expression and use the formula for the difference of two squares. $\sqrt[3]{(5\sqrt{2}+7)(5\sqrt{2}-7)}=1$ $\sqrt[3]{(5\sqrt{2})^2-7^2}=1$ $\sqrt[3]{50-49}=1$ $1=1$ Proved. Is this proof right? If not, where is the error?		detecting errors in other people's reasoning.