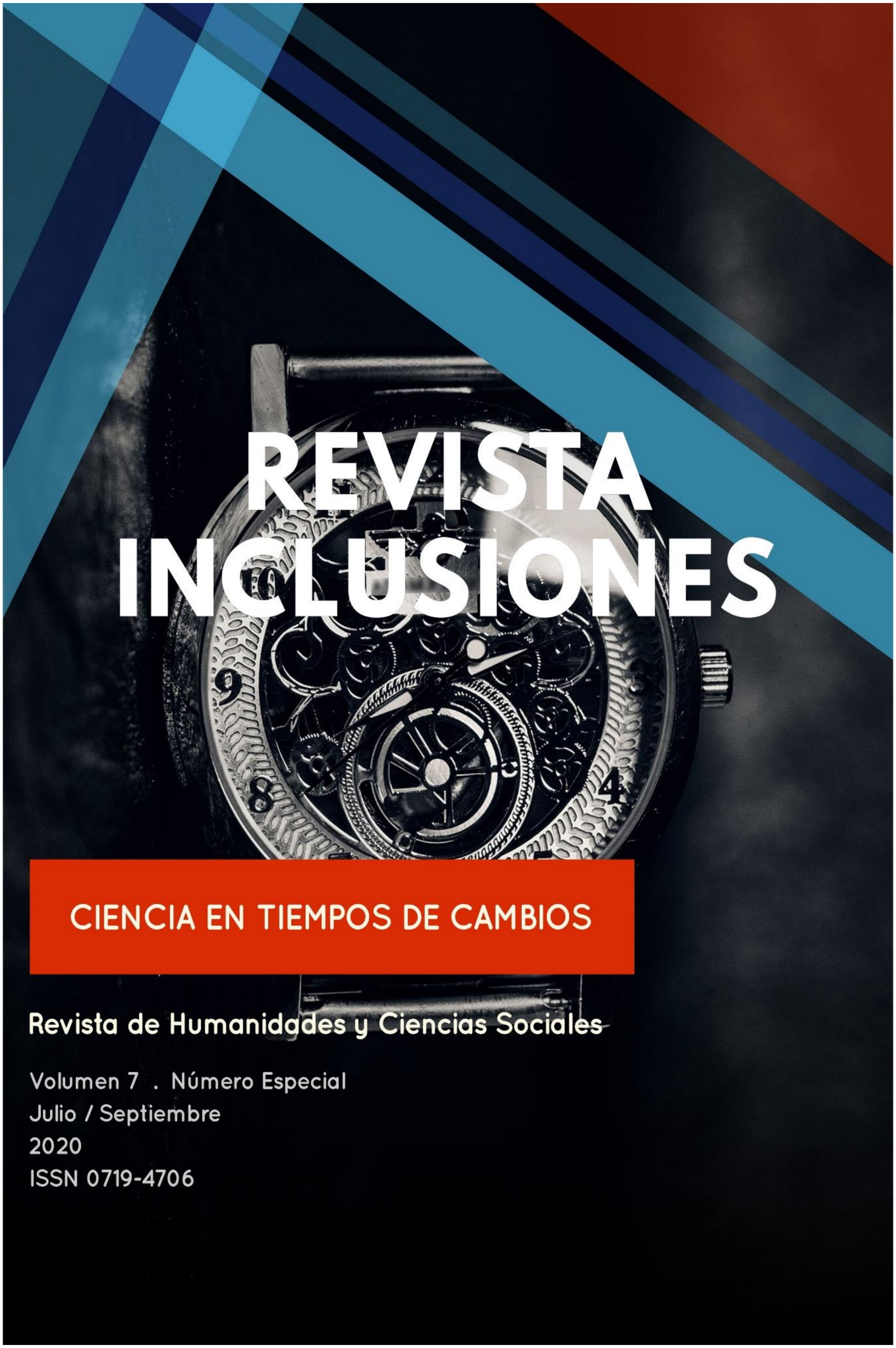

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SPATIAL LOGIC CONCEPTS

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Abstract

The purpose of this work is to study and systematize spatial logic. The present paper shows the value of spatial logic arising from its various applications. It studies the relationship between mathematical logic and spatial logic. The article shows the similarities and differences between these logics. It is proved in the study that the main advantage of spatial logic is that it makes it possible to understand the semantics, take into account the coordination of spatial objects and solve problems in the conditions of information uncertainty. The article systematizes various types of spatial logic. It describes the concepts of spatial logic that are common to the family of logics and logic-specific. The paper contains an analysis of some typical errors of figurative logic that are found in cartography. The main problems and contradictions in the development of spatial logic are singled out. The effect of granularity in figurative logic is revealed. The article points out the importance of spatial logic for artificial intelligence and robotics.

Keywords

Spatial logic – Geometric logic – Topological logic – Figurative logic – Mathematical logic

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Introduction

The knowledge that a person possesses exists in different forms. There is spatial knowledge and geo-knowledge. Spatial knowledge describes spatial images and the relationship between them. Spatial knowledge relies on geometry, artificial intelligence methods, and spatial reasoning methods[1]. Spatial logic is used to obtain spatial and geo-knowledge. Spatial logic has similarities and differences from mathematical logic. Spatial logic deals with spatial images[2]. It is based on logical reasoning, geometric laws, and topological properties. Spatial logic has its own languages. Languages of spatial logic, in contrast to the language of mathematical logic, tend to be ambiguous and uncertain. Spatial logic is applied in many areas. It is often used implicitly, without designating it as a special kind of logic. Back in the pre-Christian era, spatial logic was used in architecture and is used at present. Fibonacci series, the golden ratio, and architectural orders are examples of the use of spatial logic. Spatial logic is applied in landscape design[3]. It is used in information security systems[4]. It is used in the extraction of spatial knowledge and the analysis of ontologies[5]. Spatial logic is used in parallel computing[6] and in organizing graphs of database queries[7]. Spatial logic is used in the analysis of near-Earth space[8] and in the analysis of technical vision schemes[9]. In combination with linguistics, spatial logic is used to describe visual languages[10].

In cartography and geomatics, spatial logic is used in mapping and geodata analysis[11]. As part of spatial thinking, spatial logic is used in psychology[12]. It is applied in spatial economics[13]. The variety of applications of spatial logic makes its generalization and systematization relevant.

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[1] C. Bailey-Kellogg and F. Zhao, “Qualitative spatial reasoning extracting and reasoning with spatial aggregates”, AI Magazine, num 24(4) (2003).
[2] M. Aiello; I. Pratt-Hartmann and J. Van Benthem, What is Spatial Logic? Handbook of spatial logics, Springer, Dordrecht (2007): 1-11.
[3] E. Talen, “The spatial logic of parks”, Journal of Urban Design, Vol: 15 num 4 (2010): 473-491.
[4] S. J. Collier and A. Lakoff, “Distributed preparedness: the spatial logic of domestic security in the United States”, Environment and planning D: Society and space, Vol: 26 num 1 (2008) 7-28.
[5] D. A. Randell; Z. Cui and A. G. Cohn, “A spatial logic based on regions and connection”. KR, Vol: 92 (1992): 165-176.
[6] L. Caires, and L. Cardelli, “A spatial logic for concurrency (part I)”, Information and Computation, Vol: 186 num 2 (2003): 194-235.
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Materials and methods

The materials for the research were studies in the field of spatial analysis, psychology, cognitology, geometry and architecture, studies in the field of spatial knowledge and description of the language of spatial aggregation. Comparative analysis was used as the main method. As additional methods, system and logical analysis were used in formal and non-formal forms. The method included the differentiation of spatial logic by type of application.

Results

Mathematical logic and spatial logic

There are similarities and differences between mathematical and spatial logic. Common to mathematical and spatial logic is that they are used, first of all, for qualitative analysis. Mathematical logic and spatial logic are often used for comparative analysis. The similarity between mathematical and spatial logic is the study of relations between objects. Classical logic studies the relationship between formal or formalized objects in the space of parameters. Spatial logic studies the spatial relations between objects in real space. Mathematical and spatial logic have their own languages and units of description.

There are many differences between classical and spatial logic. Mathematical logic operates with logical variables and logical functions that have the meaning of “true”, “false”. Spatial logic operates primarily with spatial images and functions, which can have a larger set of meanings. Mathematical logic does not allow uncertainty and requires compliance with the law of excluded middle. Spatial logic allows for uncertainty and allows for non-compliance with the law of excluded middle.

Mathematical logic operates with opposition variables ($A, \neg A$), for which the double negation rule holds.

$$A = \neg(\neg A)$$

Spatial logic operates primarily with dichotomous variables [14] (A, B) for which the double negation rule does not hold.

$$(A \neq B) \wedge (\neg B \neq A) = 1$$

Spatial logic also operates with opposition variables. The operations of spatial logic are wider than the operations of mathematical logic and include the operations of mathematical logic. This leads to the fact that, within spatial logic, mathematical logic performs support functions only in some cases.

Another difference is associated with many semantic descriptions in spatial logic. One image can be interpreted in different ways. Geometrically identical graphics can have a different meaning. For example, the “black square” (Malevich), the logical square (Aristotle), the semiotic square (Pospelov), the geometric square, etc.

The difference between mathematical logic and spatial logic is due to the presence of uncertainty and modality in spatial descriptions. For example, the statement “Object A is closer to Object B than object C” can be true or false. But it cannot be described only by operators of mathematical logic. It relates to the field of spatial logic, probabilistic logic, and to qualitative reasoning[14].

Spatial logic family concept

The main concept of spatial logic is that it consists of a family of logics. Spatial logic has qualitative varieties: geometric logic, topological logic, set-theoretic logic, cartographic logic, figurative logic (the logic of technical vision, the spatial logic of artificial intelligence).

Common to the family of spatial logics is the relationship between geometry and mathematical logic, between set theory and mathematical logic, between cognitive modeling and logic.

The construction of reality depends on the basic geometry of logical structures and axioms. For example, the assumption of the sphericity of the Earth gives reason to use the geoid model to model the shape of the earth. The assumption of an ellipsoidal shape of the Earth gives reason to use the model of a common terrestrial ellipsoid for modeling the shape of the Earth.

Geometric logic concepts

Geometric logic describes an ideal world with ideal relations, but with the help of abstract figures it models real spatial objects. Geometric logic is divided into similarity logic and construction logic. Based on the geometric logic of similarity, spatial reality is modeled and models are created that have similarities with real spatial objects. The geometric logic of similarity is related to the morphology of spatial objects.

The geometric logic of construction is based on axioms (A), which differ for different geometries. Formula B is called the axiomatic following of axioms A1, A2, A3, ... An, if for any choice of the values of the variables included in A1, A2, A3, ... An, formula B takes on the value which is true when any of the formulas A1, A2, A3, An receives meaning of "truth".

$$A1, A2, A3, An \models B$$

The $\models$ sign means following. By analogy with propositional logic, rules exist in geometric logic. In geometric logic, a geometric formula A is called deducible or a theorem if there is a chaining in which the last formula equals A. Such a chaining is called a derivation of formula A. The rule can be used from the propositional calculus, which is applicable in geometric logic. Formula A is deducible from the set of formulas G if there is a chaining from G in which A is the last formula. In this case,

$$G \vdash A.$$

The $\vdash$ sign means deductibility. Thus, for geometric logic, there is the concept of logical consequence, theorem and deductibility.

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[14] J. Renz and B. Nebel, “Qualitative spatial reasoning using constraint calculi”, Handbook of spatial logics (2007): 161-215.

Photogrammetric or geodetic crossbearing are based on the rule of geometric logic for a flat triangle. If the set of plane triangles T has three internal angles *a, b, c,* then for any triangle Ti (*ai, bi, ci*) the following statement holds:

$$\forall(ai \in \mathrm{Ti},\ bi \in \mathrm{Ti},\ ci \in \mathrm{Ti},) \mid a + b + c = \pi$$

This simple logical rule defines many geodetic and photogrammetric constructions and allows solving spatial problems. The geometric construction logic is applied independently in geodesy and photogrammetry. Geometric logic includes the following components: definitions, postulates, and theorems, target statements (tasks) to be proved or constructed using some standard geometric inference mechanism. In geometric logic, geometric chaining is logical chaining.

There is the forward and backward geometric (logical) chaining. Forward logical-geometric chaining is used in the construction of complex models based on simpler ones. An example is the construction of a geographical map. The backward logical-geometric chaining is used when decomposing complex images into simple ones. For example, in automated image processing and in solving problems of object recognition, the backward logical-geometric chaining is used.

The concept of geometric logic consists in the possibility of making logical constructions by means of graphics without using the apparatus of mathematical logic.

The concept of geometric logic is the possibility of using metrics and metric spaces.

The concept of geometric logic lies in the possibility of using abstract geometric figures for spatial reasoning and qualitative comparative analysis.

The concept of geometric logic lies in the possibility of using metric models of real objects for comparative quantitative analysis.

Topological logic concepts

Topological logic complements geometric logic and is used in geo-informatics and cartography. The concept of "topology" itself can be interpreted as the "logic of place". Topological logic is a form of spatial logic. It uses logical axioms: topological invariance (Fig. 1), intersection, absence of intersection. Topological logic uses the topological properties of objects, the presence of which means "truth", while the absence means "false".

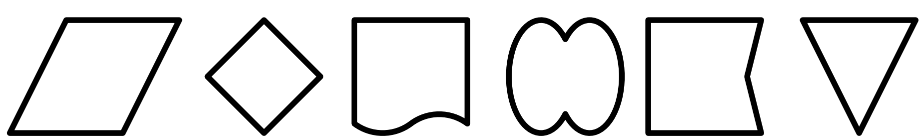

Figure 1
Logical correctness in topology. Topological invariants

The figures in Fig. 1 differ from the positions of geometric logic, but from the position of topological logic they are equivalent. They can be considered as equivalent topological figures or as equivalent topological descriptions.

In geomatics, additional conditions for the logical correctness of spatial images appear. They consist in the fact that intersections of lines should be marked, and connecting lines should not go beyond the borders.

Fig.2 shows spatial images that are correct and incorrect in geomatics and cartography. The image in Fig.2a is an example of an incorrect image from the standpoint of spatial construction logic. This spatial structure is called "spaghetti." The intersections are not marked and the ends of some lines go beyond the borders. Fig.2a represents spatial logical uncertainty.

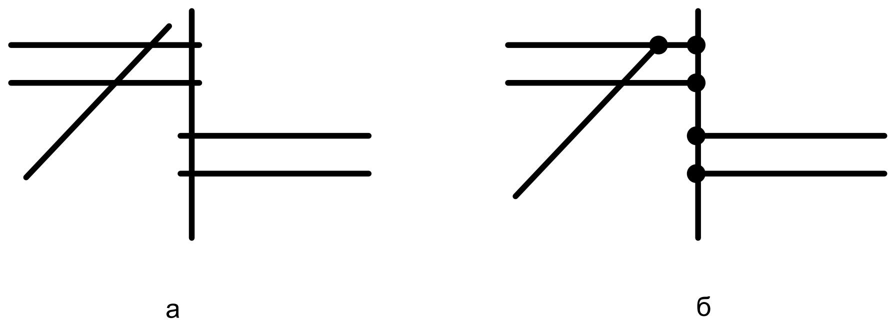

Figure 2
Topologically correct b) and topologically incorrect a) spatial image

The situation shown in Fig.2a, occurs during automated vectorization[15]. It is characterized by a violation of complementarity and topological information correspondences. Fig.2b is an example of spatial logical correctness. In Fig.2b, the errors contained in Fig.2a are corrected. Intersections are made where they are and are marked by dots. The ends of the lines that go beyond the borders are cut off. Fig.2b can be considered as logically correct from the standpoint of spatial logic. It features spatial relations. Topological invariance corresponds to logical equivalence or tautology.

The concept of topological logic consists in simplifying the consideration of abstract graphic objects by eliminating morphology and coordination.

The concept of topological logic is to simplify the modeling of spatial objects by highlighting intersections and relations. The concept of topological logic consists in considering linear objects as the main objects for studying their connections and relations.

Set-theoretic spatial logic

Set-theoretic logic is applicable to spatial objects that can be described using sets. Set-theoretic logic is subdivided into spatial and non-spatial. The spatial set-theoretic logic includes that part of the set-theoretical relations, which has spatial images as an informational correspondence. The best examples of such logic are the Euler-Venn diagrams[16].

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[15] S. Clode, et al. “Detection and vectorization of roads from lidar data”, Photogrammetric Engineering & Remote Sensing, Vol: 73 num 5 (2007): 517-535.
[16] H. Chen and P. C. Boutros, “VennDiagram: a package for the generation of highly-customizable Venn and Euler diagrams in R”, BMC bioinformatics, Vol: 12 num 1 (2011): 35.

Set-theoretic logic is used in geomatics when working with areal models, layers (groups of models) and less often with vector models. In this case, vector models are considered as linear ranges.

Set-theoretic logic uses set-theoretic relations as logically correct relations between spatial sets. In set-theoretic schemes, relations are represented, as opposed to relations in topology. Formal representations, or set-theoretic language, are associated with the spatial relations of spatial abstract images. In set-theoretic logic, morphological aspects are not considered. In the set theory, mainly relations of belonging, intersection (conjunction), and unification (disjunction) are considered.

The concept of set-theoretic logic consists in simplifying the modeling of spatial objects by highlighting only the set relations of belonging, inclusion, intersection, unification, etc.

The concept of set-theoretic logic consists in excluding from consideration the coordinate properties of the elements of the set.

The concept of set-theoretic logic consists in examining areal objects as the main objects for studying their connections and relations.

Figurative logic

Figurative logic does not describe an ideal world with ideal relations, but models the real world with real relations. Figurative logic considers spatial morphology[17] and spatial coordination[18] of objects. This property makes figurative logic the basis for use in robotics. A feature of some figurative spatial logics is the plurality of interpretation. This creates logical ambiguity. The situation of plurality of constructions leads to logical errors in the interpretation of models of spatial images.

Similarity-difference errors are attributed to the first type of figurative logic errors. These errors are caused by the use of graphic units of different semiotic types, which create similar or identical-looking image designs. In cartography, they are found only at the level of analysis of individual legends and maps (Fig. 3)[19]

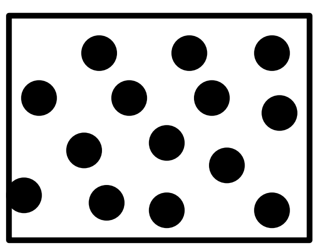

Figure 3
Duality of interpretation of spatial image

[17] A. Turner, "Analysing the visual dynamics of spatial morphology". Environment and Planning B: Planning and Design, Vol: 30 num 5 (2003): 657-676.
[18] S. Brueckner; H. V. D. Parunak, and J. Sauter, A. Spatial coordination system: patent No. 7415313 (USA, 2008).
[19] A.A. Liuty, Map language: nature, system, functions (Moscow: GEOS publishers, 2002).

The spatial model in Fig. 3 can be interpreted in two ways.

1. Fig. 3 shows an area on which the totality of separate objects represented by points is located. Such an interpretation implies that each point is a separate object and these objects are densely located. This interpretation describes a lot of objects or a heterogeneous area.

2. Fig. 3 shows an area with a symbol used for depicting the surfacing - spot filling. This is a homogeneous area, the semantic content of which is indicated by dots. This interpretation describes a single object.

The second type of spatial logical errors includes errors that arise due to incorrect placement of characters in the image field. An example is shown in Fig. 4.

a *b*

Figure 4
Error of incorrect and correct placement

In Fig.2a, an object can be interpreted either as a single object, or as two closely spaced objects. This means information uncertainty. In Fig.2b, duality of interpretation is excluded. It clearly shows that two objects are depicted. Errors of the Fig.4a type lead to the appearance of false symbolic compositions and relations, as well as to the effects of “absorption” of some signs by others. This group includes errors of semantic uncertainty and duplication of information. To eliminate such an error in cartography, the technique of “artificially spacing” the nearby objects is used. For example, on geographical maps, the embankment often shifts from the bank of the river, although in reality it is located on it.

The third group of errors in figurative spatial logic includes errors caused by mistakes in the classifications of spatial objects. For example, the symbol of a point in the form of a small circle on a geographic map can be perceived as a small area with dimensions.

Figurative spatial logic has its own languages, which are most often formed by sets of graphic primitive elements, that is, basic graphic information units[20], from which complex information units or complex models of spatial objects are designed. When analyzing spatial relations, the language of spatial aggregation is used[21] [22]. This language is an analogue of conventional cartographic signs and an example of the language of spatial figurative logic.

[20] P. A. Dokukin, “Graphic information units”, Prospects for science and education, num 3 (2015): 32-39.
[21] C. Bailey-Kellogg, F. Zhao and K. Yip “Spatial aggregation: language and applications”, AAAI/IAAI, Vol: 1 (1996): 517-522.
[22] L. Gómez, et al. “Spatial aggregation: Data model and implementation”, Information Systems, Vol: 34 num 6 (2009): 551-576.

Fig. 5 shows logical information units expressing spatial relation[23] in the language of spatial aggregation.

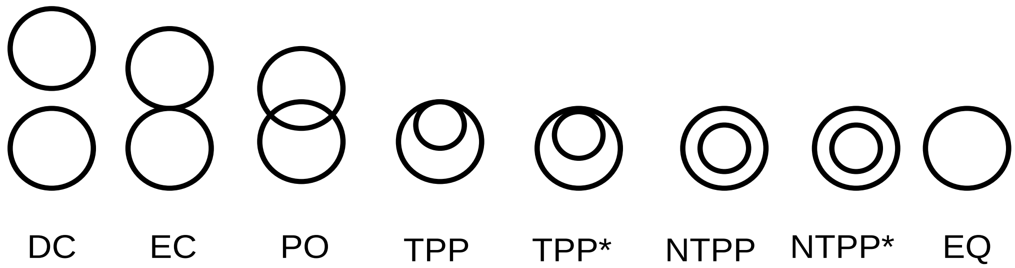

Figure 5
Logical units of spatial aggregation

In Fig. 5, the graphic symbols denote formal designations adopted in the language of spatial aggregation. They are used as logical information units, reflecting the spatial relations of two objects. In the language of spatial aggregation, the exclusion of the duality of perception of spatial images is achieved due to the difference in symbolic designations.

Spatial aggregation language (SAL) allows one to study neighborhood relations and equivalence predicates, as well as graphically examine and modify results. This language is one of the numerous languages of informatics. The initial SAL set can be downloaded from the site www.cs.purdue.edu/homes/cbk/sal.html или www.parc.com/zhao/sal.html.

A feature of figurative logic is that the spatial knowledge used by figurative logic often has a granular structure and is context-sensitive. A granule is understood as a group of objects united by the proximity of characteristics or similar functional features[24]. A granule can also be understood as a collection of objects united by some restriction. An example of a granular logical structure is a cognitive map.

The concept of figurative logic consists in the possibility of including the coordinate properties of objects (a coordinate system as the basis of logical constructions) when considering real space.

The concept of figurative logic consists in the possibility of multiple interpretations of graphic designs (geographical map), which makes it possible to describe and disclose uncertainty.

The concept of figurative logic consists in the possibility of granular construction of graphic structures (cognitive map), which makes it possible to coordinate uncertainty and certainty in a logical scheme.

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[23] C. Bailey-Kellogg, F. Zhao and K. Yip "Spatial aggregation: language and applications", AAAI/IAAI, Vol: 1 (1996): 517-522.
[24] J. Canny, et al. Granular data for behavioral targeting using predictive models: patent No. 7921069 (USA, 2011).

Discussion

A description of a complex spatial model in words creates more uncertainty than is contained in a graphic image. Graphics are often more expressive than verbal descriptions, since their interpretation involves the cognitive perception of a person and image sensors. We can say that graphics include linguistic and paralinguistic information units, and linguistics includes only linguistic units. This means that some tasks of spatial logic are untranslatable by linguistic means, but are expressed only by graphic symbols. Hence the problem of the ambiguity of the interpretation of the system "graphics - linguistics".

The general methodology of spatial logic is to find logical descriptions, including spatial or non-spatial descriptions, that can be analyzed objectively. Logical analysis methods that work with simple models are often powerless when faced with a complex, ambiguously interpreted model. The use of informative graphic images creates the advantage of spatial logic over mathematical logic.

Mathematical logic examines the logical, topological and set-theoretic relations between objects with the aim of establishing the truth, taking into account the absence of uncertainty. It does not have an apparatus that takes into account the coordination of objects.

Spatial logic explores the same relations with the addition of semantic relations and spatial coordination relations. Spatial logic explores these relations both in the absence and in the presence of uncertainty. It follows that any language that does not contain a semantic description and a description of coordinates cannot describe spatial objects and spatial relations.

The apparatus of spatial logic, in contrast to the apparatus of mathematical logic, allows one to describe semantics, take into account coordinates, and describe uncertainty.

An important difference between spatial and linguistic systems is the use of relations. Spatial models use spatial relations to directly represent domain objects. In linguistic systems, linguistic relations are used to indirectly represent the domain objects.

Spatial logic is widely used in cartography and geomatics. In these sciences, it is applied implicitly in the form of certain rules of thumb. Spatial logic is also used in geodetic surveying, photogrammetry and cartography.

Spatial logic is undergoing development and generalization. It is characterized by the absence of a unified theory and particular theories as applied to various directions.

A common concept that can be adopted for many spatial logics is the concept of spatial language. Spatial logic in a particular area must have its own logical language. The absence of a language of logic makes it unreasonable to talk about logic. The second concept is the use of information units as logical information units and as a language alphabet. Any language has an alphabet, which is formed by information units. In mathematical logic, these units are logical connectives. In spatial reasoning, such a language is, for example, the language of spatial aggregation, containing information units of spatial relations.

For spatial logic, it is reasonable to introduce the concept of “logical spatial construction”. This concept combines spatial formalism with logical formalism.

There is a problem of spatial logic, which consists in a contradiction between the expressiveness of a complex image and the complexity of its modeling by the methods of mathematical logic.

Conclusion

Unlike classical logic, spatial logic does not describe an ideal world with ideal relations, but rather models a real world with real relations. Spatial logical analysis is a growing field of research that integrates applications in different areas. The existing formal analysis of figurative representations does not always take into account the ways of using spatial relations in such representations. Set-theoretic spatial logic reflects relations without coordination. Topological logic reflects relations and connections without coordination as well. Such spatial logics complement each other when working with a set of spatial images. Figurative spatial logic takes into account coordination and semantics.

The high efficiency of spatial logic is due to the fact that its units contain semantics, while the units of mathematical logic do not contain semantics to the same extent. The semantics of spatial logic units provide the expressiveness of representation. The effectiveness of spatial logic also lies in the ability to express spatial relations by means of graphic images that are linguistically either not expressed or expressed with uncertainty.

Research in spatial logic is aimed at studying spatial relationships with the aim of using and modeling in technical systems that solve practical problems of decision-making in spatial situations, especially in robotics and in vision systems. In spatial logic, we are talking about reproducing the basic properties and relations of spatial objects. Spatial logic is not reducible to mathematical logic. This determines its importance and the need for further research.

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